

# DOCUMENT RESUME

ED 036 431

24

SE 007 748

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 TITLE A PILOT PROJECT TO DEVELOP MATERIALS FOR USE IN THE  
 PRESERVICE AND INSERVICE MATHEMATICS TRAINING OF  
 ELEMENTARY TEACHERS.  
 INSTITUTION MICHIGAN UNIV., ANN ARBOR.  
 SPONS AGENCY OFFICE OF EDUCATION (DHEW), WASHINGTON, D.C. BUREAU  
 OF RESEARCH.  
 REPORT NO CRP-S-442-66  
 BUREAU NO BR-5-8411  
 PUB DATE 67  
 CONTRACT OEC-S-551-65  
 NOTE 25P.  
 EDRS PRICE MF-00.25 HC-\$1.35  
 DESCRIPTORS \*CURRICULUM DEVELOPMENT, \*ELEMENTARY SCHOOL  
 MATHEMATICS, \*INSTRUCTIONAL MATERIALS, MATERIAL  
 DEVELOPMENT, \*MATHEMATICS EDUCATION, PROGRAM  
 DESCRIPTIONS, \*TEACHER EDUCATION

## ABSTRACT

REPORTED IS A PROJECT INVOLVING THE PREPARATION OF A  
 SELECTED SAMPLE OF NEW WRITTEN MATERIALS WHICH WOULD BE EASILY  
 READABLE BY THE POPULATION OF PRESERVICE AND INSERVICE ELEMENTARY  
 TEACHERS. THREE CHARACTERISTICS OF THE MATERIALS WERE TO BE (1)  
 READABILITY WITH A MINIMUM OF ADDITIONAL INSTRUCTIONAL HELP, (2) A  
 DISPLAY OF SOME OF THE CONNECTIONS BETWEEN MATHEMATICS AND THE REAL  
 WORLD, (3) SHOW THE PEDAGOGICAL OBJECTIVES AND USES OF EACH  
 MATHEMATICAL IDEAS AS THE FIELD AXIOMS, SETS, AND LOGIC. THE FINAL  
 UNITS PRODUCED WERE (1) AN INTRODUCTION TO RATIONAL NUMBERS BY JAMES  
 K. BIDWELL (SE 007 751), (2) AN INTRODUCTION TO THE THEORY OF NUMBERS  
 FOR ELEMENTARY TEACHERS BY ELTON D. FEOUGHER (SE 007 755), (3) BASIC  
 NUMBER AND ORDER IDEAS BY ROBERT G. CLASON (SE 007 750), (4)  
 CONCEPTUAL MODELS IN THE TEACHING OF NUMBER AND OPERATION BY ROBERT  
 G. CLASON (SE 007 754), (5) LOGICAL THINKING BY SISTER MARY DE  
 LOURDES MCALOON (SE 007 749), (6) USES OF THE FIELD PROPERTIES IN  
 ELEMENTARY SCHOOL BY ALBERT P. SHULTE (SE 007 752), AND (7) USES OF  
 MATHEMATICS IN OTHER SUBJECT AREAS BY ALBERT P. SHULTE (SE 007 753).  
 BECAUSE THIS WAS A PILOT PROJECT FOR THE DEVELOPMENT OF MATERIALS, NO  
 STATISTICAL ANALYSES OR CONCLUSIONS ARE REPORTED. (RP)

ED036431

BR 5-8411  
PA 24

A PILOT PROJECT TO DEVELOP MATERIALS  
FOR USE IN THE PRESERVICE AND INSERVICE  
MATHEMATICS TRAINING OF ELEMENTARY TEACHERS

Cooperative Research Project S-422-66  
(Small Contract Proposal S-551-65)

Phillip S. Jones  
University of Michigan  
Ann Arbor, Michigan

1965-1966

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE  
OFFICE OF EDUCATION

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The research reported herein was supported by the  
Cooperative Research Program of the Office of Education,  
U.S. Department of Health, Education and Welfare.

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### The Problem

A rapid change in the mathematics program of the elementary schools was being initiated and accelerated by such forces as the reports of the Teacher Training Panel of the Committee on the Undergraduate Program of the Mathematical Association of America and experimental textbooks of the School Mathematics Study Group. For these advocated changes to become effective both preservice and inservice teachers must be trained not only in new mathematical content, but also in new methodology and especially in the goals and in the relevancy of the new programs to these goals and to the remainder of the elementary school curriculum. This task is a tremendous one because of the numbers of teachers involved and their traditionally non-mathematical orientation.

One need was for sound mathematical materials which were so readable as to be usable in large classes and for self study. Another need was for related materials which would show the relevancy of these mathematical materials both to the teaching process and to the real world uses of mathematics.

At the University of Michigan these problems were met in four settings: (1) a large undergraduate mathematics course required of preservice teachers, (2) an inservice

institute for elementary teachers, supervisors and principals, (3) extension courses and workshops conducted in off-campus centers, and (4) summer session courses for inservice teachers and supervisors. We are still concerned with developing multi-media packages of instructional materials to improve and individualize instruction for group (1), and materials for (3) and (4).

The first step for all these groups seemed to be the development of written materials with three characteristics in addition to the sound mathematics coming out of new texts. These characteristics were: readability with a minimum of additional instructional help, a display of the continuing pedagogical connections and utility of the mathematics taught, a display of some of the connections between mathematics and the real world.

The project being reported was beginning of work upon this first step.

#### The Objectives

The purpose of this project was to prepare and polish a selected sample of new written materials which would be easily readable by the population of preservice and inservice elementary teachers. The materials were to contain not only the "new mathematics", but also new approaches and new expo-

sitions of the relationships between this mathematics and its teaching and between mathematics and other curricular areas, especially science.

### Procedures

Four advanced graduate students in mathematics education were selected to form a team with Professors J.N. Payne, A.F. Coxford, C.F. Brumfiel and P.S. Jones. The students had all had secondary school teaching experience and had worked with one or more of the four groups of preservice and inservice teachers listed under "The Problem". The graduate students were: James K. Bidwell, Robert G. Clason, Sr. Mary deLourdes McAloon, Albert P. Shulte. Bidwell was replaced by Elton E. Beougher after about one-half a year when he decided to leave campus to accept a teaching position at Central Michigan University.

The team met initially to extend and discuss the list of possible units, to formulate general criteria for their content and writing policies, to select the units to be initiated, and to schedule procedures and times for consultation, writing, and review.

The final units, writers and special consultants were:

An Introduction to Rational Numbers

James K. Bidwell (Consultant Prof. Brumfiel and,  
later, Prof. Payne)

An Introduction to the Theory of Numbers for Elementary  
Teachers

Elton D. Beougher (Consultant Prof. Jones)

Basic Number and Order Ideas

Robert G. Clason (Consultant, Prof. Jones)

Conceptual Models in the Teaching of Number and Operation

Robert G. Clason (Consultant, Prof. Jones)

Logical Thinking

Sister Mary deLourdes McAloon (Consultants, Professors  
Payne and Coxford)

Uses of the Field Properties in Elementary School

Albert P. Shulte (Consultant, Prof. Jones)

Uses of Mathematics in Other Subject Areas

Albert P. Shulte (Consultant, Prof. Jones)

Two copies of these materials, including in the case of  
the unit on Number Theory an Appendix with additional proofs,  
and a set of answers to the problems, are being submitted  
along with this report.

The preparation of each of these units involved the following stages:

1. Discussion by the entire team of its over-all purposes, suggestions for illustrations, applications, problems, and teaching aids.
2. Writing by a team member with continuing consultation with a professorial staff member.
3. Duplication of first drafts to be read by the entire team and then discussed and criticized at a team meeting.
4. Stages 2 and 3 included the preparation of a detailed list of objectives, of terms, and of content assumed as prerequisite to the unit, and of terms defined within the unit.
5. Revision by the original author with help from his consultant.
6. Duplication of a revised version for use with students.
7. At some stage after stage 3 and prior to the preparation of the final revised versions all manuscripts were read by two excellent elementary school teachers who had earlier been in our inservice institute:  
Mrs. Jenny Dubois of Ann Arbor, Michigan, and Mrs. Patricia Edmister of Dearborn, Michigan.



They not only annotated copies but joined the team discussion of the early drafts and also met individually with the authors of each unit.

They criticized the clarity of the exposition from the viewpoint of elementary school teachers and also suggested problems, comments, and even new sections to be included in the final drafts.

In some cases there was more than one preliminary duplicated draft and the point at which these teachers gave their critiques varied.

8. These units had different types of try-outs with different student populations. For example, the unit on Number Theory was used in a Saturday inservice institute for elementary school teachers, principals, and supervisors, in an extension course taught off campus, and with a group of preservice teachers. Similarly the unit on Uses of the Field Properties in Elementary School was used with all three of these groups. Both of these units had been fairly well polished before their use and seemed to be especially welcomed by the inservice teachers who appreciated the pedagogical suggestions and connections pointed out in the latter unit, and who, somewhat unrecognizedly, benefited from the rather slow, easy, detailed elementary development of

Number Theory. This latter unit appeared, in fact, to be too low in its level and too slow in its pace for the undergraduate preservice group.

On the other hand, the unit on Rational Numbers was never used with a group of students due, in part, to Mr. Bidwell's change of position and the resulting delay in his polishing of the material.

As pointed out in our Technical Progress Report on this project, several aspects of the original proposal had to be modified due to the extended delay in the final receipt of funds. We were forced to begin with no summer preparatory work, nearly a term late, with only two students for a reduced fraction of their time. These conditions led to some continuing delays as a result of which materials were not always ready at the times they were needed for classroom use. However, as will be noted later, the development of these materials has served several other, partially anticipated, purposes.

#### Conclusions and Implications

Since this was a pilot project for the development of materials, we do not have statistical analyses or conclusions to report.

The Unit on Number Theory seems to have been received as appropriate and interesting content by both inservice and pre-service teachers with the latter finding the pace and level of the writing a little low.

The Uses of the Field Properties in Elementary School was well received as a summarizing survey of the mathematical importance and pedagogical uses of these properties by teachers who had been exposed to a straightforward mathematical treatment. Appendix A to this report contains a cover page for this unit which shows the analysis of background and objectives such as was done for all units. This also shows the textbook series analyzed as a preparation for writing the unit. Appendix B is a table of contents for the unit. Mr. Shulte (now Dr. Shulte) also made an interesting survey of the occurrence and maintainance of these concepts in a number of elementary school textbook series. This is summarized in an appendix to his unit which I hope he will sometime have time to expound in an article which I believe would be useful to both text writers and text selectors.

Mr. Shulte's other unit, Uses of Mathematics in Other Areas, is potentially important at this time because of the renewed emphasis upon teaching mathematics for its applicability and also because of the development of new curriculum materials in other areas which make use of mathematics.

When Dr. Shulte can prepare an article for publication based

on his unit he will perform a service. His previous training and experience as a science teacher was particularly useful here. He also made contact with some new programs in the social sciences.

All of the graduate students who worked on this project have now completed Ph.D. programs in mathematics education. I feel that the "bull sessions", arguments within the team and consultations with faculty members contributed significantly to their interest and maturation as Ph.D. candidates and mathematics educators.

In three cases, those of Bidwell, Clason, and Sr. Mary deLourdes McAloon, their theses were very closely related to the work they did in preparing materials for this project. Sr. deLourdes conducted a study on teaching units in logic in grades three and six using twenty-five teachers and eleven hundred students in the schools of the Archdiocese of Detroit. She wrote logic units for the students and gave inservice training to the teachers.

Mr. Bidwell conducted a comparative experimental study of several different ways of teaching rational numbers.

Mr. Clason made a historical study of the changing approaches to the concepts of cardinal and rational numbers in American schools as they were effected by new developments in mathematics, psychology, and philosophy.

Sr. deLourdes is presently consulting with a commercial publisher with reference to weaving a thread of logic into an elementary school textbook series. Dr. Shulte is now directing an extensive Oakland County project in developing a mathematics course for disadvantaged and non college bound students.

The University of Michigan staff is working on the development of further materials, perhaps films and tapes, to improve our preservice course for elementary teachers.

From my biased viewpoint the value of the returns from this project to improved education far exceed the dollar value invested. Its effects have been and are continuing. As noted earlier, no statistical measures were projected in our planning of this pilot project in the development of materials. However, the analyses and conclusions submitted above are supported by the individual written and oral reactions obtained from the experienced elementary school teacher critical readers who met with the team, and also by the written and group oral reactions of students in both preservice and inservice classes who used some of the materials.

Appendix A

COVER PAGE--USES OF THE FIELD PROPERTIES IN ELEMENTARY SCHOOL

Albert P. Shulte

U.S.O.E. Elementary Materials Project

I. Background assumed

- A. Previous exposure to the field properties. This does not imply thorough understanding, but does assume some familiarity with the terminology and concepts.
- B. Basic knowledge of the operations and algorithms used with the whole numbers, the fractional numbers (rational numbers) and the fractional numbers expressed in decimal form.

II. Placement of the Unit

Because of the above assumptions, the unit should be placed after an introduction to the field properties. It could follow such an introduction immediately, as an extension of such a unit, or it could be used somewhat later, to provide review, extension, and a spiral approach to the topic of field properties.

III. Objectives for the Unit

- A. To show teachers places where the field properties are

used in developing the arithmetic of the elementary school.

- B. To give teachers a greater command of the field properties.
- C. To provide teachers with more practice in the use of the field properties.
- D. To provide teachers with information that will be useful when pupils, parents, or other teachers question the worth of teaching one or another of the field properties.
- E. To help teachers to gain deeper insight into the structure of the number systems studied in elementary school arithmetic.

#### IV. Books Surveyed for Writing the Unit

SMSG, Grades K-6

American Book Company (Deans, et. al.), Grades 1, 3-6

Modern Arithmetic Through Discovery (Silver Burdett), 1-6

GCMP, Grade 1, and Intermediate Series (1964-65), Booklets  
1-12, Grades 4-6

Seeing Through Arithmetic (Scott-Foresman), Grades 1-6

Moving Ahead in Arithmetic (Holt, Rinehart, Winston),  
Grades 1-6



Elementary School Mathematics (Addison-Wesley), Grades 1-5

Elementary Mathematics (Harcourt, Brace, & World), Grades

1-3



Appendix B

CONTENT OUTLINE--THE USES OF THE FIELD PROPERTIES IN ELEMENTARY  
SCHOOL

U.S.O.E. Elementary Materials Project

Albert P. Shulte

1. Introduction

- a. Introductory remarks
- b. Table presenting the field properties

2. Addition and Subtraction of Whole Numbers

2.1 Manipulation of Concrete Objects

2.2 "Ringin'" Sets

2.3 Regrouping

2.4 Adding in Different Orders

2.5 Addition Tables

2.6 The Number Line

2.7 A General Rearrangement Property

3. Multiplication of Whole Numbers

3.1 The Multiplication Algorithm

3.2 Multiplying by Multiples of 10

3.3 Special Multiplication Algorithms

- a. "Criss-cross" Multiplication

- b. Lattice Multiplication

3.4 Doubling a Product

- 3.5 Arrays and Rectangular Regions
- 3.6 Concrete Objects
- 3.7 Multiplication Tables
- 4. Division of Whole Numbers
  - 4.1 The "Stacking" Algorithm
  - 4.2 Distributivity of Division Over Addition
  - 4.3 The distributive Property and the Euclidean Algorithm
  - 4.4 Dividing a Product by Two
- 5. Comparing and Contrasting Addition with Subtraction and  
Multiplication with Division
- 6. Renaming Fraction and Mixed Numbers
- 7. Addition and Subtraction of Fractional Numbers
- 8. Multiplication of Fractional Numbers
- 9. Division of Fractional Numbers
  - 9.1 The Reciprocal of a Number
  - 9.2 Reinterpreting Division as Multiplication
  - 9.3 The Complex Fraction Technique
  - 9.4 The Common Denominator Technique

10. Fractional Numbers Represented in Decimal Form

10.1 Addition and Subtraction of Decimals

10.2 Multiplication of Decimals

10.3 Division of Decimals

11. Integers and Rational Numbers

11.1 Additive Inverses

11.2 Addition and Subtraction of Positive and Negative  
Numbers

11.3 Multiplication of Positive and Negative Numbers

12. Miscellaneous Uses of the Field Properties

12.1 Measures

12.2 Abstract Operations and Operation Tables

12.3 Applications

12.4 Mental Arithmetic

13. Conclusion

a. Concluding remarks

b. Chart showing places in the unit where specific  
field properties are used

c. Chart showing the grade range where the topics are  
studied.

Appendix C

Bibliography for  
USES OF MATHEMATICS IN OTHER SUBJECT AREAS

Science

- ScA Concepts in Science, Brandwein, Cooper, Blackwood, Hone; Harcourt, Brace, and World, 1966. Grades 1, 2, 4, 5, 6.
- ScB Watching and Wondering, Minnemast, 1964.
- ScC Science for Work and Play 1, Science for Here and Now 2, Science Far and Near 3, Science in Your Life 4, Herman and Nina Schneider; Heath, 1965.
- ScD Science, Silver Burdett, 1965. Grades 1-6.
- ScE Science 4--Extending Your Knowledge. Silver Burdett, 1966.
- ScF Science Activities from A to Z, Dr. Helen Challard and Helen Brandt. Children's Press, 1963.
- ScG Science--A Process Approach, AAAS, 1964. Commentary for Teachers and Parts One--Six.
- ScH Science Curriculum Improvement Study, Robert Karplus, University of California, Berkeley.
- (MO) Material Objects, 1963.
- (VM) Variation and Measurement, 1964.
- (IS) Interaction and Systems.
- (RPM) Relativity of Position and Motion
- (S) Solutions.
- ScI 1: Science is Fun, 2: Science is Learning, Scott, Foresman, 1965 and 1961 respectively.

ScJ Exploring Science, Thurber and Durkee, Allyn and Bacon, 1964. Grades One, Two, Three, Five.

ScK Today's Basic Science, Navarra and Zaffaroni. Harper and Row, 1963. Grades 1-5.

### Social Studies

SSA Basic Social Studies. Row, Peterson, and Company, 1958. Grades 2 and 3.

SSB Basic Social Studies. Harper and Row, 1964. Grades 4 and 5.

SSC Heath, 1964.

(ANH) A New Hometown.

(ISO) In School and Out.

(GUSA) Greenfield, U.S.A.

(CAW) Communities at Work.

SSD In These United States and Canada. Heath, 1965.

SSE Knowing Our Neighbors in Canada and Latin America. Holt, Rinehart, and Winston, 1965.

SSF Living in the Old World. The Macmillan Company, 1961.

SSG Our United States in a World of Neighbors. Holt, Rinehart, Winston, 1964.

SSH The Basic Social Studies Program. Scott, Foresman, 1965.

(AH) At Home.

(AS) At School.

(ITN) In The Neighborhood.

(ICTC) In City, Town, and Country.

(IAOS) In All Our States.

(ITA) In the Americas.

(BTA) Beyond the Americas.

Economics

- EA Developmental Economic Education Program (DEEP), Joint Council on Economic Education, 1964.
- Part One--Economic Ideas and Concepts.
- Part Two--Suggestions for Grade Placement and Development of Economic Ideas and Concepts
- EB Economic Education, M.L. Frankel. The Center for Applied Research in Education, Inc., New York City, 1965.
- EC Economic Education Experience for Enterprising Teachers, Vol. 3, 1964-65. Kazanjian Foundation Awards, 1966.
- ED Economic Education in the Schools, Report of the National Task Force on Economic Education, September, 1961.
- EE Economic Education Topics as suggested by the National Task Force CED. 711 Fifth Avenue, New York 22, N.Y. (Mimeographed sheet distributed by the Michigan Council on Economic Education).
- EF Economics and the Consumer, Joint Council on Economic Education, 1966.
- EG Some Suggestions for Economics in the Elementary School, Dr. George L. Ferber, Associate Director, Joint Council on Economic Education. (Mimeographed sheet).
- EH Suggested Basic Areas in Teaching Economic Education. (Lithoprinted sheet by the Michigan Council on Economic Education).
- EI The History of Money. A work sheet for introducing economic education into the curriculum, prepared by Lois Walter, a fourth grade teacher.

People

- PA Mrs. Dubois, reader-critic, U.S.O.E. Elementary Materials Writing Project.
- PB Mrs. Edmister, reader-critic, U.S.O.E. Elementary Materials Writing Project.

- PC Dr. Theral T. Herrick, Executive Director of the Michigan Council on Economic Education.
- PD Professor Waldo Tobler, Department of Geography, The University of Michigan.
- PE Donald Beard, Assistant to the Director, Institute for Economic Education.